Wellbore Performance: Mathematical Modeling and Analysis

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Abstract
Wellbore performance analysis involves establishing a relationship between tubular size, wellhead and bottomhole pressure, fluid properties and fluid production rate. As fluid flow through the wellbore there is a drop in pressure which tend to impede production rate. The modified Bernoulli's equation was applied in calculating the acceleration pressure drop and frictional pressure drop in a horizontal wellbore in which the flow is turbulent. The plot of pressure drop against flow rate was obtained which was similar to that obtained by several authors. Several plots were generated for different sizes of tubing to determine the effect of tubing size on the production rate.

Keywords: *Horizontal wellbore, Mathematical modeling, Pressure drop, and Wellbore performance*

Introduction

The achievable oil production rate from a well is determined by wellhead pressure and the flow performance of production string that is; tubing, casing and both, the flow performance of production string and properties of fluids being produced and the fluid in the wellbore (i.e., oil, water, gas) and sand. Several analytical and experimental works have been publish to determine the performance of both horizontal and vertical wellbore as fluid flow through them. Understanding the factors responsible for the pressure drop as the fluid is produced is very vital.

The analysis of the production performance is essentially based on the following fluid and well characteristics:

a) Fluid PVT properties
b) Relative permeability data
c) Inflow-performance-relationship (IPR) (Ahmed, (2006))

The flow in the wellbore is either single-phase or multiphase. In most production wells, the flow is multiphase, with at least two phases (e.g. gas and liquid) present. Some production wells and most injection wells experience single phase flow.

The flow geometry of interest in the wellbore is generally flow through a circular pipe, though flow in an annular space such as between tubing and casing sometimes occurs. Pressure drop is a huge challenge in the petroleum industry as it impedes flow rate and the production potential of the well.

Asheim and AL. E, (1992) proposed the first semi analytical model to evaluate the performance of a horizontal well with the consideration of the wellbore-pressure drop resulting from turbulent flow. They made a study on friction factor correlation for horizontal wellbore, which included acceleration pressure drop caused by the continuous fluid inflow along the wellbore.

They stated that both wall friction factor and radial influx acceleration contributed to the total pressure drop along the perforated pipe and the pointed out the wall friction factor could be calculated the same way as for a regular and unperforated pipe geometrically similar to a wellbore casing (Yue el a, 2014). Su and Gudmundsson (2014) conducted a set of single-phase experiment in a perforated pipe with radial inflow. In these experiments, water is used as the working fluid.

Su and Gudmundsson, (1995) showed that most of the pressure drops in the pipe is due to friction and acceleration effect. In certain case studies the pressure drop along the wellbore was studied just by considering only the frictional component. In most circumstances, the pressure drop is studied taking the acceleration into consideration by neglecting the other effects like inflow, mixing etc. Asheim and AL. E, (1992) stated that the total pressure drop along a perforated pipe is made up of wall friction and inflow acceleration and computed the wall friction factor in the same way for regular, unperforated pipe.
With the increase in the flow velocity, the momentum influences the pressure drop in addition to the friction pressure drop. This part of the pressure drop has been addressed by several authors in recent years. Abdulwahhab et al, (2014) made a theoretical study of pressure drop in a partially perforated wellbore. The various factors that contribute to the total pressure drop in a perforated pipe was determined theoretically. In addition to the pressure profile along a blank section downstream of a perforated section were measured, and new wall-friction-factor correlation for pipe flow with perforation influx were calculated.
In this study, we analyze the effect of wall friction on fluid flow in the production tubing using the modified Bernoulli's equation for turbulent flow for a horizontal pipe by considering the pressure drop due to friction and acceleration. Several correlations has been developed to calculate friction factor. The objective of this paper is to determine the effect of tubing size on the production performance of the well.

Model Description
Assumption:

1. The horizontal section of the tubing is considered
2. The fluid flowing through the tubing is considered to be single-phase incompressible
3. The flow is turbulent

The model to be applied will be derived from the Bernoulli's equation. The equation is the most famous in fluid mechanics. Its significance is that when the velocity increases, the pressure decreases, and when the velocity decreases, the pressure increases. The Bernoulli's equation is a statement derived from the conservation of energy and the work-energy ideas that come from Newton's laws of motion as cited by Beggs, (2003). According to Fekete, (2013), the relationship between pressure and velocity in an inviscid incompressible flow was enunciated in the form of Bernoulli's equation, first presented by Euler.

$$p + \frac{1}{2}\rho V^2 = constant \qquad (1)$$

The equation in it originally form does not consider frictional pressure drop and is meant for steady state flow. Since it was also applied to a vertical tube the equation becomes;

$$p + \rho g h + \frac{\rho V^2}{2} = constant \qquad (2)$$

This equation can be further modified to consider the pressure drop due to the pipe wall friction. According to the shell intensive training manual, the modified Bernoulli's equation as it considers pressure drop due to friction is presented as;

$$p + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 + \rho g h_f \quad (3)$$

$$P_1 - P_2 = \frac{1}{2}\rho(V_1^2 - V_2^2) + \rho g(z_1 - z_2) + \Delta p_f$$

Since we considering an horizontal pipe and there is no elevation eq. becomes

$$P_1 - P_2 = \frac{1}{2}\rho(V_1^2 - V_2^2) + \Delta p_f \quad (4)$$

In fluid flow, it is convenient to work with an average velocity which remains constant in incompressible flow when the cross section of the pipe is constant. There considering the average velocity within of the pipe the equation becomes;

$$\Delta P = \frac{1}{2}\rho V^2 + \Delta p_f \quad (5)$$

The pressure due to friction, Δp_f can be calculated using Darcy-Weisbach equation
$\Delta p_f = frictional\ pressure\ drop$
And is further expressed as;

$$\Delta p_f = \frac{2f\rho L V^2}{D} \quad (6)$$

Putting eq. 5 into 4

$$\Delta P = \frac{1}{2}\rho V^2 + \frac{2f\rho L V^2}{D} \quad (7)$$

Since we are considering an incompressible fluid where the density is constant

$$\Delta P = \rho\left(\frac{q^2}{2A^2} + \frac{2fLV^2}{D}\right) \quad (8)$$

Considering a pipe with constant diameter,
Area, $A = \frac{\pi}{4}D^2$
Substituting for A in equation Beggs, (2003) we have;

$$\Delta p = \frac{8\rho}{\pi^2 D^4} q^2 \left(1 + \frac{2fL}{D}\right) \quad (9)$$

Equation indicate the total pressure drop which consist of two components;

- The pressure drop due to kinetic energy change (acceleration effects).
- The frictional pressure drop due to wall friction in the rough pipe.
  When the relative roughness of the pipe is known, an accurate and convenient relationship for the friction factor in the turbulent pipe flow is the Nikuradse equation

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\left(2\frac{\varepsilon}{D}\right) \quad (10)$$

Where

$\varepsilon = pipe\ absolute\ roughness\ factor, ft$
$\frac{\varepsilon}{D} = pipe\ relative\ roughness$
Equation can only be applied to turbulent flow

Description of the Simulator

The simulator to be used to derive the plot of the pressure drop against flow rate is MATLAB® (matrix laboratory). It is an interactive program for numerical computation and data visualization. It is used extensively by engineers for analysis and design. MATLAB® allow matrix manipulation, plotting of functions and data, implementation of algorithms, creation of user interfaces.

The software generates the pressure drop values from the flow rate values imputed using the derived model equation and as well generate the result from the plot of pressure drop against flow rate.

Results and Discussion

The result of the numerical computation using MATLAB® is presented in this work. Different classes of tubing with their respective diameter and different flow rate data were used in the computations. Total pressure drop was calculated for a rough pipe with turbulent flow using the flow rate data. The value of the pressure drop in a section of the pipe was plotted against the different flow rate data and the result presented. The result gotten is presented in graphical form. In order to verify the accuracy of the model, synthetic and real data were used for the simulation. These data include; fluid density, tubing sizes and flow rate data.

The plot obtained using synthetic data with flow rate data obtained from Beggs, (2003) with light crude of density 871kg/m^3 (0.37475lb/ft^3) is presented in figures 1 and 2. Several class of steel pipes with roughness factor ($\varepsilon = 0.00015$) were considered with constant diameter. The pressure drop of each class was calculated using the derived model by substituting the flow rate values into the equation (equation 9), with values of pipe diameter, fluid density, length of pipe also substituted to calculate the pressure drop. The calculated pressure drop values were plotted against the flow rate values. The results obtained are shown in the figures below.

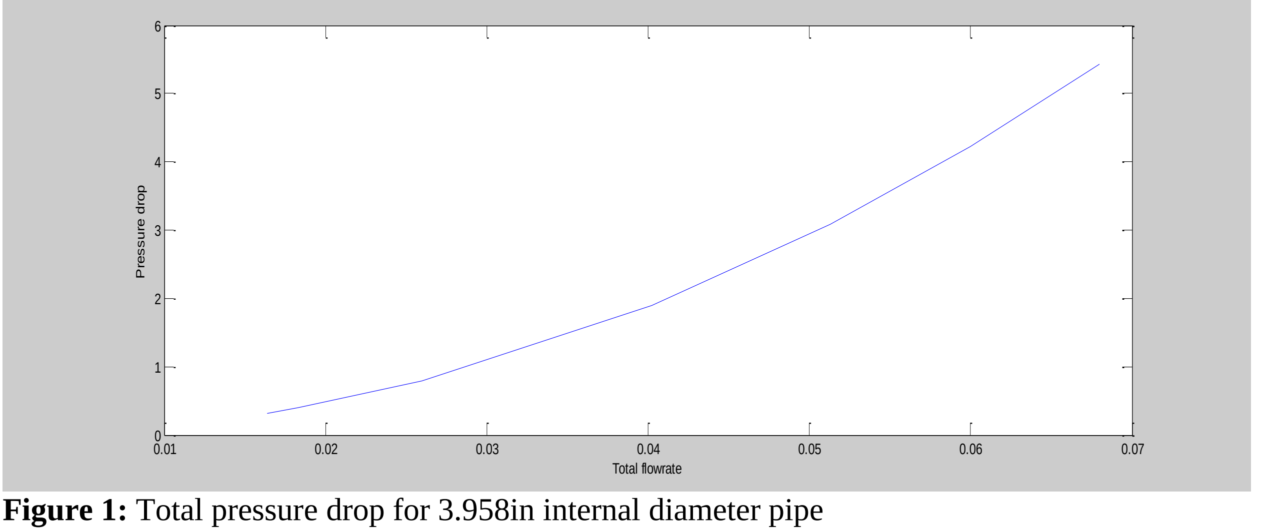

Figure 1: Total pressure drop for 3.958in internal diameter pipe

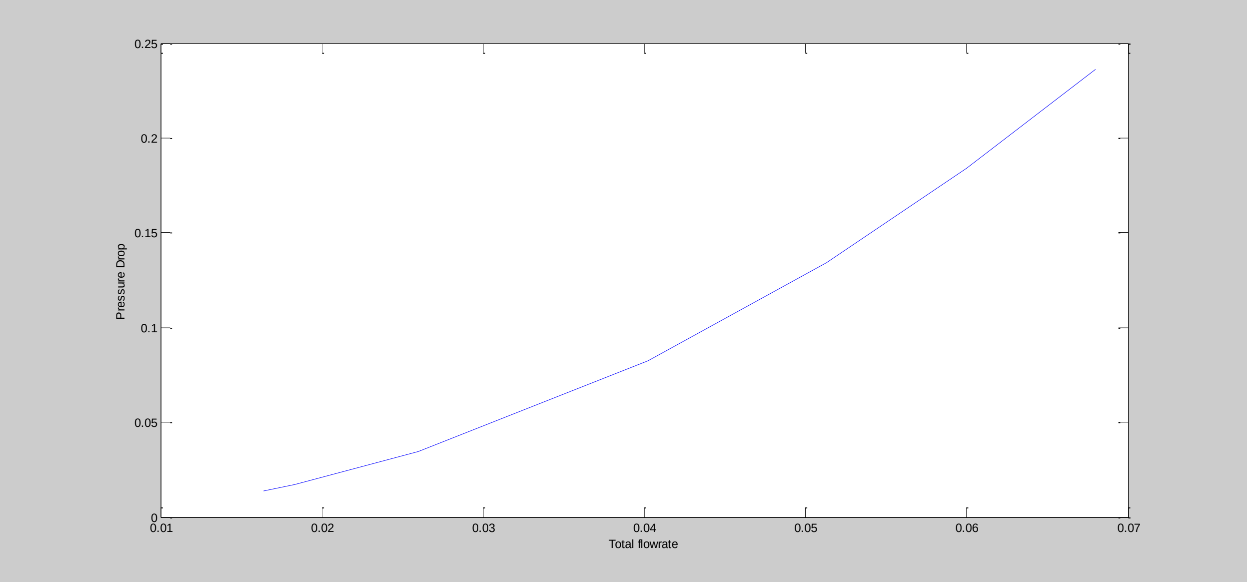

Figure 2: Total pressure drop for 2.041in internal diameter pipe

The plots shown in figures 1 and 2 are obtained using flow rate data from Beggs, (2003). This plot follow similar trend as obtained by other authors. From the plots, increasing the flow rate causes an increase in pressure drop. This shows that flow rate is directly proportional to pressure drop.

Similar plots were done using data obtained from well TXZ. This is to further verify the accuracy of the model. The plot obtained is presented in figure 3 which follows the same trend as the one obtained in figures 1 and 2

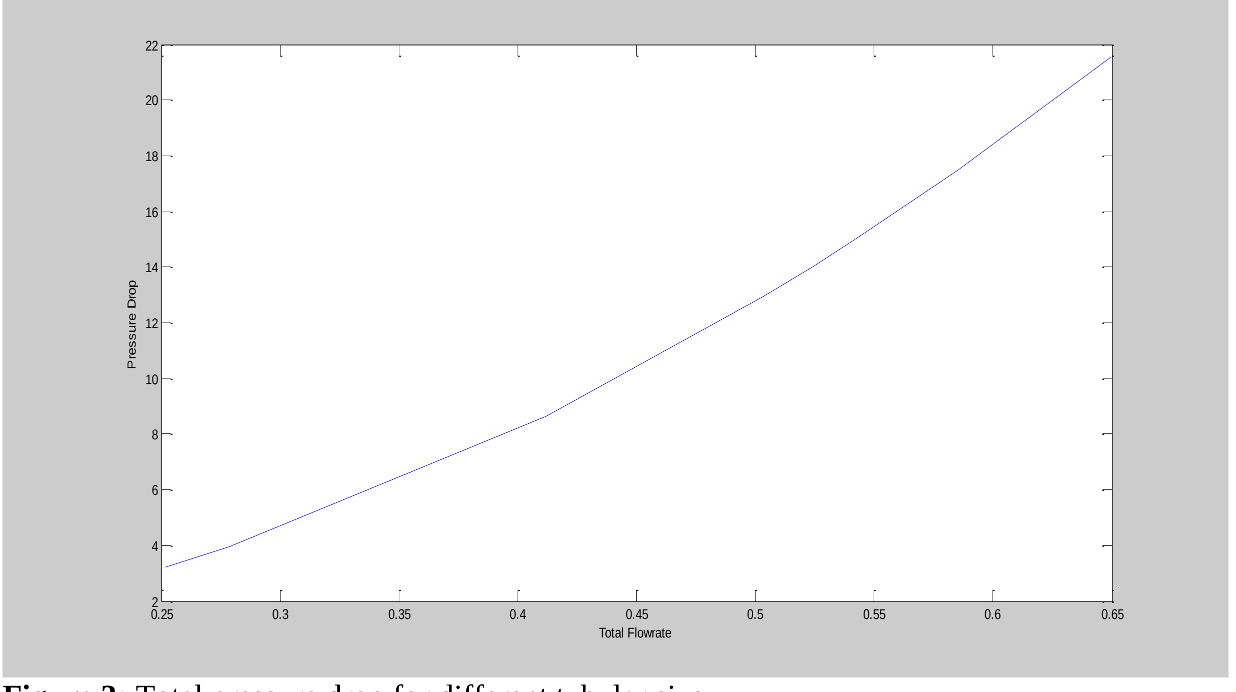

Figure 3: Total pressure drop for different tubular size

The plots obtained indicate that increasing the flow rate through the tubing will result increase in pressure drop. This is also a function of the internal diameter of the production tubing and

also the length of the production conduit. In order to address this menace of pressure drop during production, there is need to use tubing of appropriate size for optimum production.
The total pressure drop for three different pipes of different diameter were calculated and the result presented in figure 4, 5 and 6. This is to determine the effect of tubing size on production rate and how it affects the production potential of the well. The pipes are denoted with their diameters as d1, d2 and d3.

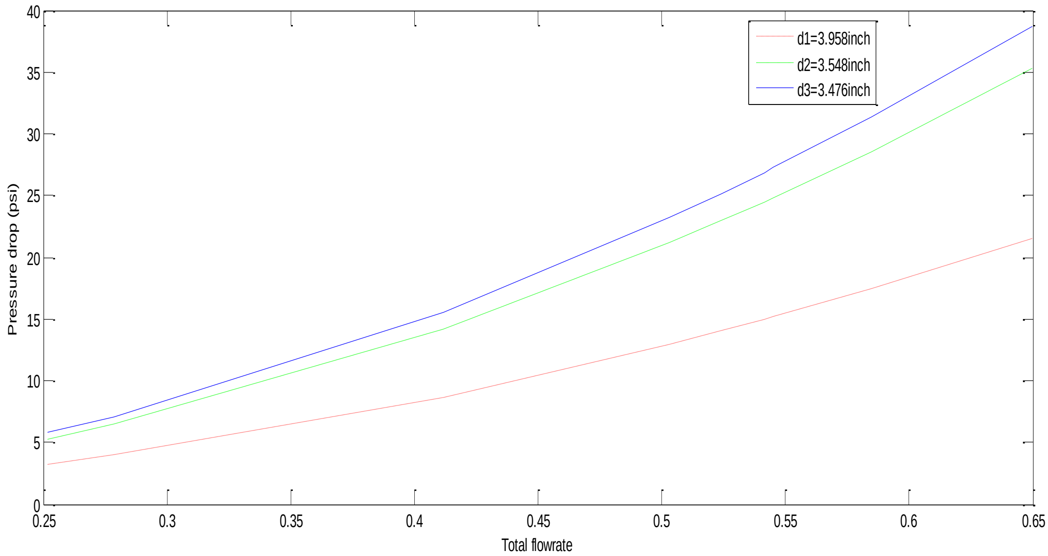

Figure 4: Total pressure drop for rough pipes with different internal diameter

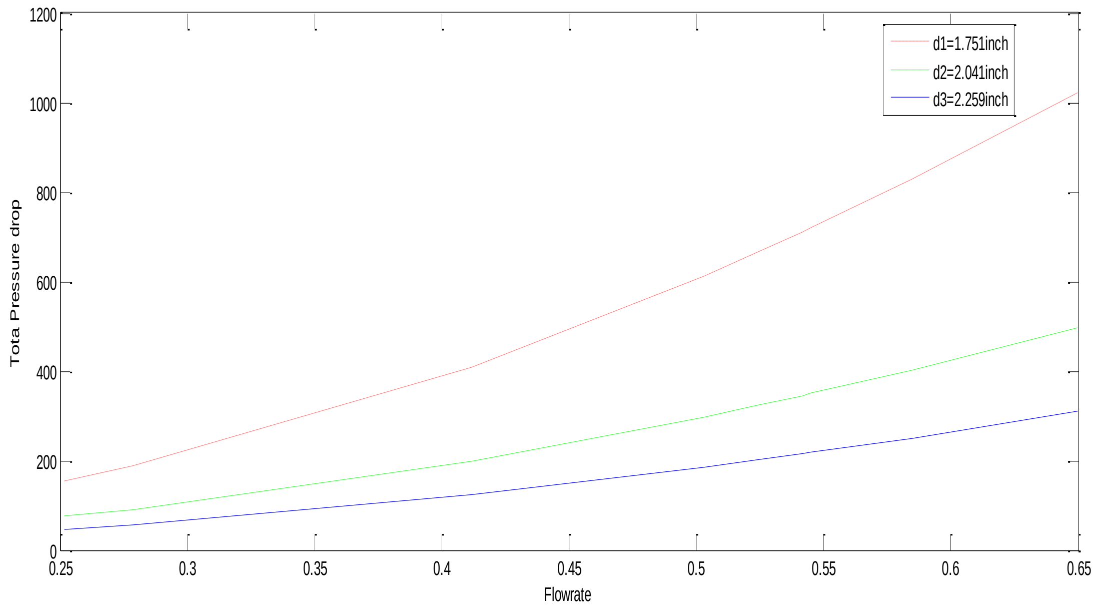

Figure 5: Pressure drop for different tubing size

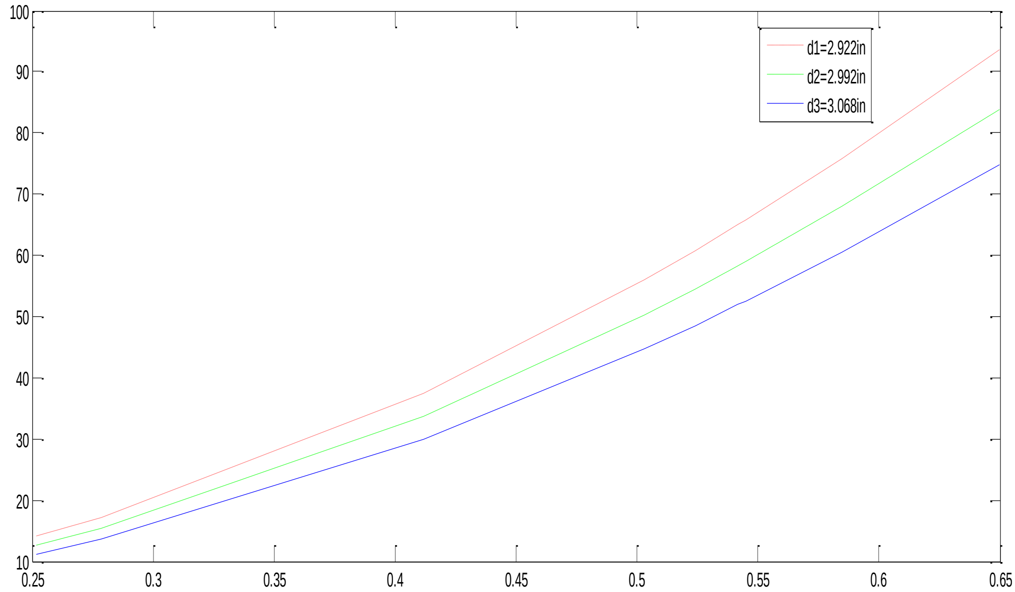

Figure 6: Pressure drop for different tubing

The plot obtained from Figures 4, 5 and 6 indicate that fluid flow rate through pipes can be influenced by the tubing size. From the plot, increasing the pipe diameter causes a reduction in pressure drop, thereby improving the production potential of the well. From this, the total pressure drop was found to be higher for smaller diameter pipe than larger diameter pipes. Therefore in order to minimize pressure drop through the tubing during production and for optimum production, a production tubing of larger diameter should be used. This will assist in optimizing production and reduce the effect of pressure drop.

Conclusion

Numerical simulations have been carried out on the fluid flow in the wellbore especially through the tubing. The total pressure drop in the wellbore is due to change in momentum (acceleration), wall friction, perforation roughness and fluid mixing. Compared with the models presented in literatures, the model generated in this research is realistic because it makes it possible to calculate the pressure drop along horizontal wellbore considering both pressure drop due to kinetic energy and friction. Horizontal well represent a long wellbore where well pressure throughout the wellbore is constant. Some pressure drop from the tip of the horizontal wellbore to the producing end to maintain fluid flow within the wellbore. If the pressure drop through the wellbore is significant as compared to the reservoir drawdown, the reservoir drawdown along the well length would also change. To calculate the changing production rate along the well length, the pressure drop along the pipe can be calculated using the model generated. Also for optimum production, production tubing of adequate size should be used to minimize pressure drop effect.

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